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Non-linear parametric identification using model reduction based MCRE and MDKF

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Résumé — This article essentially addresses the numerical frugality of model updating procedures for non-linear material behaviour using reduced order modelling. A proper generalised decomposition based formulation has been introduced that separates the governing equations into sundered space and time problems, providing low fidelity approximations. A proper orthogonal decomposition based model reduction method has also been introduced to tackle variable Hooke's tensor due to softening behaviour. A newly formulated Kalman filter is also introduced in this article for sequential model updating procedures.
Mots clés — Model reduction, Inverse problems, Kalman filtering.

1 Introduction

In the context of structural health monitoring, it is essential to detect defects early on and track their growth. This necessitates the combination of numerical simulations and physical measurements, which gives rise to dynamic data driven application system (DDDAS) [1]. The coupling between measured data and constitutive modelling is generally performed by using the data to identify the parameters of the constitutive model. The parametric identification strategy essentially involves model updating based on the adjustment of the model parameters that reduces in some manner the difference between the model prediction and the measurement data [2]. In this research, the inverse analysis method used is the modified constitutive relation error (MCRE), as it can handle partial and noisy data. MCRE is a hybrid approach, relying on both physics-based and data-based information [3].

The target problem being material non-linearity (like visco(plasticity) and damage), the resolution of the MCRE procedure incorporates an iterative procedure similar to the large time increment (LATIN) method [4]. The intent herein is to propose a proper generalised decomposition (PGD) based reduced order model (ROM) in space and time to be integrated in the LATIN type methodology for the MCRE process. The advantage of PGD is numerical cost effectiveness as ROMs provide low dimensional approximations for high fidelity models [5]. Also, for materials that can be damaged, a proper orthogonal decomposition (POD) based ROM strategy is used for approximating the variable Hooke's tensor.

Finally, this paper also deals with real time sequential data assimilation and model updating. Kalman filter (KF) is one of the most popular algorithms for data assimilation, with different available variants [7]. A new version termed modified dual Kalman filter was developed by coupling MCRE-based model updating algorithm to a non-linear dual (unscented Ka) UKF (unscented Kalman filter) leading to an online data assimilation strategy [6]. UKF was developed based on the unscented transform, resulting in a reduced size in sampling point sets compared to other KF extensions. This particular version of MDKF (modified dual Kalman filter) is used in this article which contains the integrated ROM based MCRE method, for sequential real-time data assimilation, and dynamic model updating.

2 MCRE procedure

For the development of the inverse problem, consider the domain Ω subjected to a body force field $\underline{f}_d^v$ as shown in fig. 1. It consists of a boundary $\partial\Omega$, divided into three separate boundaries. Dirichlet boundary condition $\underline{u}_d$ is applied on $\partial_u\Omega$, Neumann boundary condition $\underline{f}_d^s$ on $\partial_f\Omega$, and $\partial_\emptyset\Omega$ contains no boundary information. Also, there exist a sub-domain Ω_m such that $\Omega_m \subseteq \Omega$, which represents the region where the kinematic data is measured.

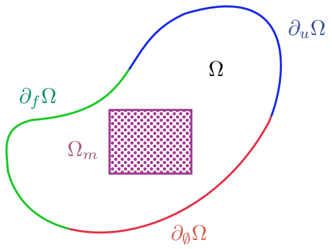

FIGURE 1 – Representative schematic of the identification problem

The Dirichlet boundary condition is defined by

$$\underline{u}(t) = \underline{u}_d(t) \text{ on } \partial_u\Omega, \text{and} \forall t, \tag{1}$$

and the Neumann boundary condition by

$$\boldsymbol{\sigma} \cdot \underline{n} = \underline{f}_d^s, \text{on } \partial_f\Omega, \text{and} \forall t. \tag{2}$$

The equilibrium equation is defined by

$$\nabla \cdot \boldsymbol{\sigma} + \underline{f}_d^v = 0, \text{in } \Omega, \text{and} \forall t, \tag{3}$$

and the kinematic compatibility by

$$\boldsymbol{\varepsilon} = \nabla^s \underline{u}, \text{in } \Omega, \text{and} \forall t. \tag{4}$$

Here, t refers to the time, $\underline{u}$ to the displacement, $\boldsymbol{\sigma}$ to the Cauchy stress, $\boldsymbol{\varepsilon}$ to the infinitesimal strain. Considering small deformations, the total strain $\boldsymbol{\varepsilon}$ is additively decomposed to elastic strain $\boldsymbol{\varepsilon}^e$ and plastic strain $\boldsymbol{\varepsilon}^p$. The operator ∇^s is defined such that $\nabla^s \circ = 1/2\left(\nabla \circ + (\nabla \circ)^T\right)$

The constitutive behaviour can be written as

$$\boldsymbol{\sigma} = \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}^e}, \boldsymbol{Z} = \frac{\partial \psi}{\partial \boldsymbol{X}}, Y = -\frac{\partial \psi}{\partial D}, \dot{\boldsymbol{\varepsilon}}^p = \frac{\partial \phi^*}{\partial \boldsymbol{\sigma}}, \dot{\boldsymbol{X}} = -\frac{\partial \phi^*}{\partial \boldsymbol{Z}}, \dot{D} = \frac{\partial \phi^*}{\partial Y}. \tag{5}$$

Here, D is the damage variable and its thermodynamic force (dual) Y is the strain energy release rate. The additional internal variables that describe the state of the material (especially hardening) is represented by the set of all the primal variables $\boldsymbol{X}$ and their associated duals $\boldsymbol{Z}$. $\partial\psi$ is the free energy function and its dual counterpart is the pseudo free energy function $\partial\psi^*$. Similarly $\partial\phi$ is the dissipation potential funtion its dual counterpart is the pseudo dissipation potential function $\partial\phi^*$. These functions are parametrised by the set of parameters $\underline{\theta}$.

The MCRE aims to identify the parameters $\underline{\theta}$ of constitutive relations that fit the best to the experimental data ,considered here to be the measured displacement $\underline{\tilde{u}}$, such that

$$\underline{u}(t) = \underline{\tilde{u}}(t) \text{ in } \Omega_m, \text{and} \forall t. \tag{6}$$

The general idea of the MCRE-type minimisation is to split the governing equations following the reliability of information. In MCRE procedure, eqs. (3) and (4) are considered to be reliable (to be satisfied) and eqs. (5) and (6) to be unreliable (to be minimised). The boundary informations (eqs. (1) and (2)) herein are considered to be reliable. This defines the MCRE functional as

$$\zeta^2 = \int_0^T \int_\Omega \eta_\psi d\Omega dt + \int_0^T \int_0^t \int_\Omega \eta_\phi d\Omega d\tau dt + \frac{\gamma}{2} \int_0^T \int_{\Omega_m} \|\underline{u} - \underline{\tilde{u}}\|^2 d\Omega_m dt, \tag{7}$$

where γ is a scaling factor. For the setting of the scaling factor, it is first represented in terms of a known coefficient and an unknown exponent. The selection of the exponent can be obtained using either the Morozov's discrepancy principle (MDP) or the L-curve principle [8]. In case of MDP, the exponent is obtained as the smallest number such that the final discrepancy between the computed and measured system response is within the noise level of the data. The L-curve criterion (used in this research) is a graphical tool (where knowledge of the noise is not required) suggests to choose the point on the curve (the CRE term and the gap term are the axes) closest to the origin.

The residual errors are defined as

$$\eta_\psi = \psi(\varepsilon^e, \boldsymbol{X}) + \psi^*(\boldsymbol{\sigma}, \boldsymbol{Z}) - \boldsymbol{\sigma} : \varepsilon^e - \boldsymbol{Z} \cdot \boldsymbol{X} - YD \tag{8a}$$

$$\eta_\phi = \phi\left(\dot{\varepsilon}^p, -\dot{\boldsymbol{X}}\right) + \phi^*(\boldsymbol{\sigma}, \boldsymbol{Z}) - \boldsymbol{\sigma} : \dot{\varepsilon}^p + \boldsymbol{Z} \cdot \dot{\boldsymbol{X}} - Y\dot{D} \tag{8b}$$

The MCRE concept essentially involves minimising the MCRE functional ζ to find the parameters $\underline{\theta}$ of the constitutive model, i.e.

$$\underline{\theta}_{opt} = \min_{\underline{\theta}} \left(\min_{\varkappa \in \mathcal{A}_{ad}} \zeta^2(\varkappa, \underline{\theta}) \right) \tag{9}$$

where $\mathcal{A}_{ad}$ is the admissibility space (of static and kinematic admissibility), and the set of the state variables $\varkappa = \{\varepsilon^e (= \varepsilon - \varepsilon^p), \varepsilon^p, \boldsymbol{\sigma}, \boldsymbol{X}, \boldsymbol{Z}, D, Y\}$.

The minimisation is performed sequentially and iteratively. For a given iteration i :

— The first step (also referred as the basic problem) involves computing the admissible state $\varkappa$ for a given parameter set $\underline{\theta}_{i-1}$, i.e.

$$\varkappa_i = \min_{\varkappa \in \mathcal{A}_{ad}} \zeta^2\left(\varkappa, \underline{\theta}_{i-1}\right). \tag{10}$$

— The second step (also referred as the identification problem) involves computing the updated material parameter by minimising the MCRE for the previously calculated admissible set, i.e.

$$\underline{\theta}_i = \min_{\underline{\theta}} \zeta^2(\varkappa_i, \underline{\theta}). \tag{11}$$

In the proposed approach, the identification problem of the inverse analysis is the minimisation of the MCRE functional (ζ^2) with respect to parameters. A convenient approach is to use the optimal admissible set, thereby it is natural to solve the minimisation problem using the gradient descent approach. The update of the parameters is given as

$$\underline{\theta}_i = \underline{\theta}_{i-1} - \hbar \frac{d\zeta^2(\varkappa_i, \underline{\theta})}{d\underline{\theta}}. \tag{12}$$

Here, $\hbar$ is the step-size of the gradient descent. The gradient is computed through the adjoint-state approach and eq. (12) can be rewritten as

$$\underline{\theta}_i = \underline{\theta}_{i-1} - \hbar \frac{\partial\zeta^2(\varkappa_i, \underline{\theta})}{\partial\underline{\theta}}. \tag{13}$$

3 The basic problem using ROM

The basic problem is the most computationally expensive part, and the resolution is performed with a strategy similar to the large time increment (LATIN) method. This is a non-incremental iterative method, which generally starts with an approximation of the quantities of interest for the whole space-time domain. Thereafter, at each iteration, the quantities of interest are improved till a convergence is reached. The MCRE functional ζ^2 is split into two parts,

$$\zeta_\psi^2 = \int_0^T \int_\Omega \eta_\psi d\Omega dt + \frac{\gamma}{2} \int_0^T \int_{\Omega_m} \|\underline{u} - \underline{\tilde{u}}\|^2 d\Omega_m dt, \text{and}\, \zeta_\phi^2 = \int_0^T \int_0^t \int_\Omega \eta_\phi d\Omega d\tau dt, \tag{14}$$

which are then minimised alternatively. The strategy starts with an elastic initialisation and thereafter a two step algorithm is used where at the global step the equations global in space are solved and in the local step equations local in space are solve.

The global step involves minimising ζ_ψ^2 under admissibility constraints. The kinematic admissibility is enforced in the discretised space as essential boundary condition. The static admissibility is enforced through a Lagrangian as

$$\mathcal{L} = \zeta_\psi^2 - \int_0^T \left[\int_\Omega \boldsymbol{\sigma} : \varepsilon(\underline{\lambda}) d\Omega - \int_\Omega \underline{f}_d^v \cdot \underline{\lambda} d\Omega - \int_{\partial_f \Omega} \underline{f}_d^s \cdot \underline{\lambda} dS \right] dt, \tag{15}$$

where $\varepsilon(\underline{\lambda})$ is the strain associated to the Lagrange multiplier $\underline{\lambda}$. A displacement field $\underline{v}$ (defined up to a rigid body motion) is introduced by duality such that the corresponding stress $\boldsymbol{\sigma}_v$ is statically admissible.

This therefore defines the variables associated to $\underline{v}$ as $\varkappa_v$ and $\underline{u}$ as $\varkappa_u$. During the global step, internal variables $\boldsymbol{X}_u$, $\boldsymbol{X}_v$, $\boldsymbol{Z}_u$, $\boldsymbol{Z}_v$, ε_u^p, ε_v^p, D_u, D_v, Y_u, Y_v are frozen to the value obtained at the last local step.

Consider a particular LATIN iteration j, the quantities $\kappa = \left\{\underline{u}, \underline{v}, \underline{\lambda}, \varepsilon_u, \varepsilon_v, \varepsilon_\lambda, \varepsilon_u^p, \varepsilon_v^p\right\}$ are represented in terms of corrections, i.e. $\Delta\kappa_j = \kappa_j - \kappa_{j-1}$. Using these corrective terms, the stationarity of eq. (15), i.e. search of the saddle point, results into the following set of equations

$$\begin{aligned}
&\int_0^T \int_\Omega \mathrm{C}_{v\,j}\left[\left(\varepsilon_{v\,j-1} - \varepsilon^p_{v\,j-1}\right) + \left(\Delta\varepsilon_{u\,j} - \Delta\varepsilon^p_{u\,j}\right) + \Delta\varepsilon_{\lambda\,j}\right] : \varepsilon\left(\delta\underline{\lambda}\right) d\Omega dt \\
&- \int_0^T \int_\Omega \underline{f}_d^v \cdot \delta\underline{\lambda} d\Omega dt - \int_0^T \int_{\partial_f\Omega} \underline{f}_d^s \cdot \delta\underline{\lambda} dS dt = 0,
\end{aligned} \tag{16a}$$

$$\begin{aligned}
&\int_0^T \int_\Omega \Big[\mathrm{C}_{u\,j}\left(\varepsilon_{u\,j-1} - \varepsilon^p_{u\,j-1}\right) - \mathrm{C}_{v\,j}\left(\varepsilon_{v\,j-1} - \varepsilon^p_{v\,j-1}\right) + \left(\mathrm{C}_{u\,j} - \mathrm{C}_{v\,j}\right)\left(\Delta\varepsilon_{u\,j}\right. \\
&\left.- \Delta\varepsilon^p_{u\,j}\right) - \mathrm{C}_{v\,j}\Delta\varepsilon_{\lambda\,j}\Big] : \varepsilon\left(\delta\underline{u}\right) d\Omega dt + \gamma\int_0^T \int_{\Omega_m} \left(\underline{u}_{j-1} + \Delta\underline{u}_j - \underline{\tilde{u}}\right) \cdot \delta\underline{u} d\Omega_m dt.
\end{aligned} \tag{16b}$$

Here, the free energy function ψ is considered to be

$$\psi = \psi_\varepsilon + \psi_X, \text{with}\, \psi_\varepsilon = \frac{1}{2}\left(\varepsilon - \varepsilon^p\right) : \mathrm{C}\left(1 - D\right)\left(\varepsilon - \varepsilon^p\right), \tag{17}$$

where ψ_X is the free energy function related to the hardening terms, C is the Hooke's tensor, and

$$\mathrm{C}_{v\,j} = \mathrm{C}\left(1 - D_{v\,j}\right), \mathrm{C}_{u\,j} = \mathrm{C}\left(1 - D_{u\,j}\right). \tag{18}$$

Now, to incorporate, the idea is to separate the space-time problem into segregated space and time problems. This is essentially done by representing the quantities of interest into separated variable forms (as functions of space and time). Consider that the variables $\underline{\lambda}_{j-1}$ and $\underline{u}_{j-1}$ were approximated as

$$\underline{\lambda}_{j-1} = \underline{\lambda}_0 + \sum_{i'=1}^{\varpi-1} \overline{\underline{\Lambda}}^{i'}\left(\Omega\right) \alpha_\lambda^{i'}\left(t\right), \underline{u}_{j-1} = \underline{u}_0 + \sum_{j'=1}^{\varsigma-1} \overline{\underline{U}}^{j'}\left(\Omega\right) \alpha_u^{j'}\left(t\right), \tag{19}$$

at LATIN iteration $j-1$ with $\underline{\lambda}_0$ and $\underline{u}_0$ being solutions of the elastic initialisation, ϖ and ς are the numbers of modes, $\overline{\underline{\Lambda}}, \overline{\underline{U}}$ represent space functions, and α_λ, α_u represent time functions. There are two ways the corrective terms $\Delta\underline{\lambda}_j$ and $\Delta\underline{u}_j$ can be calculated. The first approach is to add another space-time mode, i.e.

$$\Delta\underline{\lambda}_j\left(\Omega, t\right) = \overline{\underline{\Lambda}}^{\varpi}\left(\Omega\right) \alpha_\lambda^{\varpi}\left(t\right), \Delta\underline{u}\left(\Omega, t\right) = \overline{\underline{U}}^{\varsigma}\left(\Omega\right) \alpha_u^{\varsigma}\left(t\right). \tag{20}$$

It is possible to add more than one mode per iteration, however here it is restricted to one. Incorporating this separation in eq. (16), the space-time problem set can be separated into a set of time problem and a set space problem. The space problem and the time problem are solved through a fixed point iteration, initialised by arbitrary time functions. This approach is called expansion of the bases and the total quantities are become

$$\underline{\lambda}_j = \underline{\lambda}_0 + \sum_{i'=1}^{\varpi} \overline{\underline{\Lambda}}^{i'}\left(\Omega\right) \alpha_\lambda^{i'}\left(t\right), \underline{u}_j = \underline{u}_0 + \sum_{j'=1}^{\varsigma} \overline{\underline{U}}^{j'}\left(\Omega\right) \alpha_u^{j'}\left(t\right), \tag{21}$$

The second approach is to update the bases, which essentially involves updating the time functions with the existing spatial bases. In this case the corrective terms are represented as

$$\Delta\underline{\lambda}_{j-1} = \sum_{i'=1}^{\varpi-1} \overline{\underline{\Lambda}}^{i'}\left(\Omega\right) \Delta\alpha_\lambda^{i'}\left(t\right), \Delta\underline{u}_{j-1} = \sum_{j'=1}^{\varsigma-1} \overline{\underline{U}}^{j'}\left(\Omega\right) \Delta\alpha_u^{j'}\left(t\right), \tag{22}$$

where $\left\{\Delta\alpha_\lambda^{i'}\right\}_{i'=1}^{\varpi-1}, \left\{\Delta\alpha_u^{j'}\right\}_{j'=1}^{\varsigma-1}$ are corrections to the time functions, such that

$$\alpha_{\lambda\,\mathrm{up}}^{i'} = \alpha_\lambda^{i'} + \Delta\alpha_\lambda^{i'}, \alpha_{u\,\mathrm{up}}^{j'} = \alpha_u^{j'} + \Delta\alpha_u^{j'}, \text{with}\, i' = \{1, \ldots, \varpi - 1\}\, \text{and}\, j' = \{1, \ldots, \varsigma - 1\}. \tag{23}$$

The updated functions are represented by $\left\{\alpha^{i'}_{\lambda\mathrm{up}}\right\}_{i'=1}^{\varpi-1}, \left\{\alpha^{j'}_{u\mathrm{up}}\right\}_{j'=1}^{\varsigma-1}$. This type of separation essentially transforms the space-time problem into a time problem with known space functions, which is comparatively inexpensive to solve. The total quantities of interest for the updating approach become

$$\underline{\lambda}_j = \underline{\lambda}_0 + \sum_{i'=1}^{\varpi-1} \overline{\underline{\Lambda}}^{i'}(\Omega)\,\alpha^{i'}_{\lambda}(t), \underline{u}_j = \underline{u}_0 + \sum_{j'=1}^{\varsigma-1} \overline{\underline{U}}^{j'}(\Omega)\,\alpha^{j'}_{u}(t), \tag{24}$$

where the time functions are updated functions as per eq. (23).

Although updating the bases in inexpensive, however sometimes it is necessary to expand the bases. A criterion based on the stresses $\boldsymbol{\sigma}_u$ and $\boldsymbol{\sigma}_v$ is calculated at each LATIN iteration, which if less than a predefined value, new space-time modes are added in the next iteration.

The strains $\Delta\varepsilon_{v\,j}$, $\Delta\varepsilon_{\lambda\,j}$ corresponding to $\Delta\underline{u}_j$ and $\Delta\underline{\lambda}_j$ can be calculated from the kinematic compatibility, and the displacement $\Delta\underline{v}_j$ along with its corresponding strain $\Delta\varepsilon_{v\,j}$ can be obtained from

$$\int_0^T \int_\Omega \mathrm{C}_{v\,j}\left[\left(\Delta\varepsilon_{v\,j} - \Delta\varepsilon^p_{v\,j}\right) - \left(\Delta\varepsilon_{u\,j} - \Delta\varepsilon^p_{u\,j}\right) - \Delta\varepsilon_{\lambda\,j}\right] : \varepsilon\left(\delta\underline{v}\right) d\Omega dt = 0, \tag{25}$$

which is also a resultant of the stationarity of eq. (15).

Now, the finite element operators are defined as

$$\begin{aligned}
\mathbb{K}_{v\,j}(t) &= \underbrace{\left[\int_\Omega \mathbb{B}^T\mathrm{C}\left(1 - D_{v\,j}(\Omega,t)\right)\mathbb{B}d\Omega\right]}_{\text{assembled}}, \mathbb{K}_{u\,j}(t) = \underbrace{\left[\int_\Omega \mathbb{B}^T\mathrm{C}\left(1 - D_{u\,j}(\Omega,t)\right)\mathbb{B}d\Omega\right]}_{\text{assembled}},\\
\mathbb{L}_{v\,j}(t) &= \underbrace{\left[\int_\Omega \mathbb{B}^T\mathrm{C}\left(1 - D_{v\,j}(\Omega,t)\right)d\Omega\right]}_{\text{assembled}}, \mathbb{L}_{u\,j}(t) = \underbrace{\left[\int_\Omega \mathbb{B}^T\mathrm{C}\left(1 - D_{u\,j}(\Omega,t)\right)d\Omega\right]}_{\text{assembled}},
\end{aligned} \tag{26}$$

where $\mathbb{B}$ is the derivatives of the shape functions, $\mathbb{K}$ is the stiffness matrix and $\mathbb{L}$ is a similar matrix operator that projects through integration the strains at Gauss points to the nodes. Now, due to the presence of damage, these operators are time dependent and require computation at each time step and expensive integration in time for solving the space problem. To mitigate this bottleneck, singular value decompositions (SVDs) are performed on the snapshots $1 - D_{u\,j}$ and $1 - D_{v\,j}$. From which ι and μ modes are selected such that

$$1 - D_{u\,j} = \sum_{i^*=1}^{\iota} \overline{D}^{i^*}_u \alpha^{i^*}_{d_u}, 1 - D_{v\,j} = \sum_{j^*=1}^{\mu} \overline{D}^{j^*}_v \alpha^{j^*}_{d_v}, \tag{27}$$

where $\overline{D}_u$ and $\overline{D}_v$ are the space functions, and α_{d_u}, α_{d_v} are the time functions. It has to be mentioned that the number of modes to be selected is based on the magnitude of the singular values. As $\iota, \mu \ll$ the number of time steps, the calculations and integrations of these finite element operators are drastically reduced. The resolution of the space problem is made frugal by using SVD.

In the local step, the minimisation of ζ^2_ϕ corresponds to the integration of evolution laws at each Gauss point. The integration of the evolution laws is performed with an Euler scheme with the total strains ε_u and ε_v frozen to the values obtained from the last global step. The initial conditions on internal variables are enforced for the first time step, and the solution strategy is similar to an elastic predictor-(visco)plastic corrector type methodology. In the rate-independent case the dissipation potential is the indicator of a convex domain and is not differentiable, whereas in the rate-dependent case, this potential is differentiable.

The stopping criteria (for the iterative process) is essentially defined through a LATIN error indicator defined using the total strains after each global step, and if this indicator is less than a pre-defined value, the LATIN iterations are terminated.

4 Modified dual Kalman filter

Now, to interface the numerical model with the on-the-fly observations, sequential data assimilation is necessary. In the context of parametric identification, it means model updating time step by time step. This can be achieved through the procedures described before, with subdivision of the time interval and performing the inverse analysis sequentially through each of the subdivisions. However, for sequential performance, the process will be expensive, as the expensive basic problem needs to be solved several times per time step.

To circumvent this difficulty, a popular methodology for sequential data assimilation, the Kalman filter is used here. The system generally consists of two equations, namely a state prediction equation, and an observation equation. Kalman filters follow a prediction-correction scheme consisting of two steps. In the prediction step, the system state at the next stage is estimated. This estimation is modified at the second (correction) step by new measurements acquired. The dual Kalman filter (DKF) is an useful extension of the Kalman filtering appropriate for inverse problems, where the parameters are kept in the state vector and the observation operator is turned into a state evolution operator based on a second Kalman filter.

In case of MCRE process, the admissible state being computed nonetheless, can be combined with the dual Kalman filter quite effectively. The key idea of the combined procedure is to use the admissible fields of the MCRE method as the dual observations in the dual Kalman filter. This results in what is referred to as the modified dual Kalman filter (MDKF).

5 Numerical example

For the numerical verification of the methodologies described before, a classical two-dimensional plate with a hole problem is considered (see fig. 2). The material behaviour considered is elasto-viscoplasticity coupled with isotropic hardening and isotropic damage. The material parameters consist of the Young's modulus E, the Poisson's ratio ν, the yield stress σ_y, R_∞ dictating the isotropic hardening, the viscous coefficient K and exponent n, the damage threshold Y_{th}, damage coefficient S and damage exponent s.

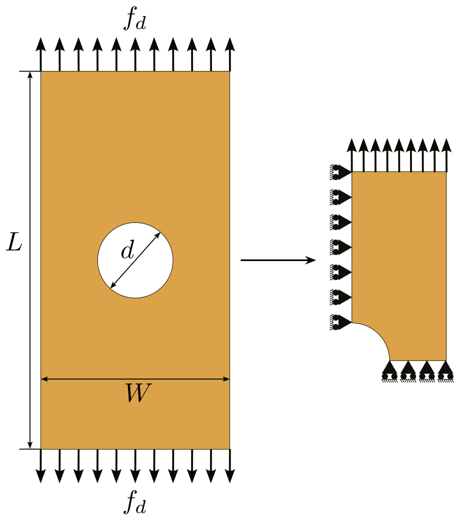

FIGURE 2 – A rectangular plate with circular hole under traction

The measurement data $\tilde{\underline{u}}$ is synthetically generated by solving the direct problem. To check the accuracy of the ROM, reference basic problem resolutions (without any reduced order modelling) are performed using the reference material parameters,and then ROM based formulations are used to solve the basic problem. The CPU time for ROM based resolution was found to be 70% of the classical resolution. The displacement fields $\underline{u}, \underline{v}$ depict errors of $6.3 \times 10^{-13}\%, 1.2 \times 10^{-11}\%$ with respect to reference. The stresses $\boldsymbol{\sigma}_u, \boldsymbol{\sigma}_v$ have errors of $1.5 \times 10^{-11}\%, 5.5 \times 10^{-10}\%$. The errors corresponding to the plastic strains $\varepsilon_u^p, \varepsilon_v^p$ are obtained to be $2.9 \times 10^{-11}\%, 4.9 \times 10^{-11}\%$, and $1.6 \times 10^{-11}\%, 6.9 \times 10^{-10}\%$ for the total strains $\varepsilon_u, \varepsilon_v$. The errors for r_u, r_v are obtained to be $2.8 \times 10^{-11}\%, 4.8 \times 10^{-11}\%$, and their corresponding duals R_u, R_v provide the same error values. Errors corresponding to D_u, D_v are $1.3 \times 10^{-11}\%, 1.7 \times 10^{-11}\%$, and errors for Y_u, Y_v are $2.6 \times 10^{-11}\%, 8.3 \times 10^{-10}\%$. 15 PGD modes were generated to approximate both $\underline{u}$ and $\underline{\lambda}$, and 8 SVD modes to approximate both D_u, D_v. From these analyses, it was seen that that admissible states provide excellent accuracy for the ROM based resolutions of the basic problem, compared to the classical resolutions. Also the ROM based resolutions were found to be frugal compared to the classical resolutions.

Next is to to solve the total inverse problem, with the elastic parameters (E,ν) considered to be known. Each of the material parameters shown in fig. 3 are identified individually, with the rest conside-

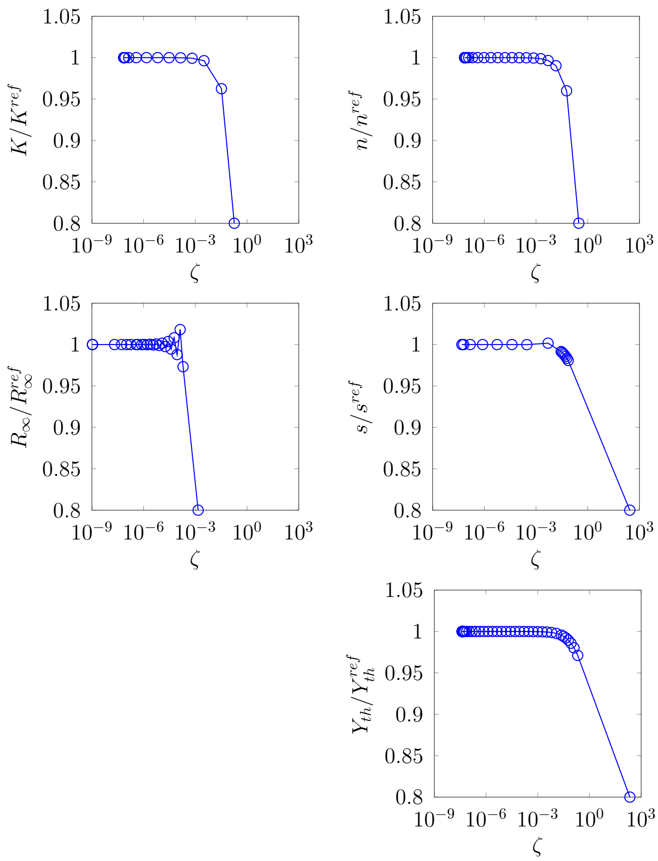

FIGURE 3 – The convergence curve of material parameters with respect to the MCRE

red to be known and equal to their reference values. It is clear from fig. 3 that the parameters converge to their reference values with successive iterations of the gradient descent algorithm, and the MCRE functionals decrease to their minimal values.

The next exercise is to perform parametric model updating using MDKF and compare with the MCRE procedure, as depicted in fig. 4. Again, each of the material parameters shown in fig. 4 are identified individually, with the rest considered to be known and equal to their reference values. It has to be noticed that the parameters in fig. 4 remain unaltered at their initialised values for the several initial time steps. This is due to the fact that these initial time steps correspond to elastic loading, and the inelastic material parameters will only be updated after the onset of inelasticity and not before that. The usefulness of MDKF is its frugality, and MDKF time was 28% of MCRE time for n, 35% for K, 25% for σ_y, 30% for R_∞ and s, 31% for S and Y_{th}.

6 Conclusion

The accuracy and frugality of PGD based reduced order models for the resolution of MCRE based inverse approaches for non-linear model updating are depicted here. A POD based model reduction method has also been coupled, to treat the variable Hooke's tensor efficiently. The methodology has been tested on elasto-viscoplastic-damage material identifications, and the veracity along with numerical frugality has been observed to be appreciable. Also, a sequential model updating method using a modified dual Kalman filter has been used which has proved to be extremely frugal compared to the classical MCRE process. The overall attempt to introduce ROM in MCRE and thereby in MDKF has been quiet successful.

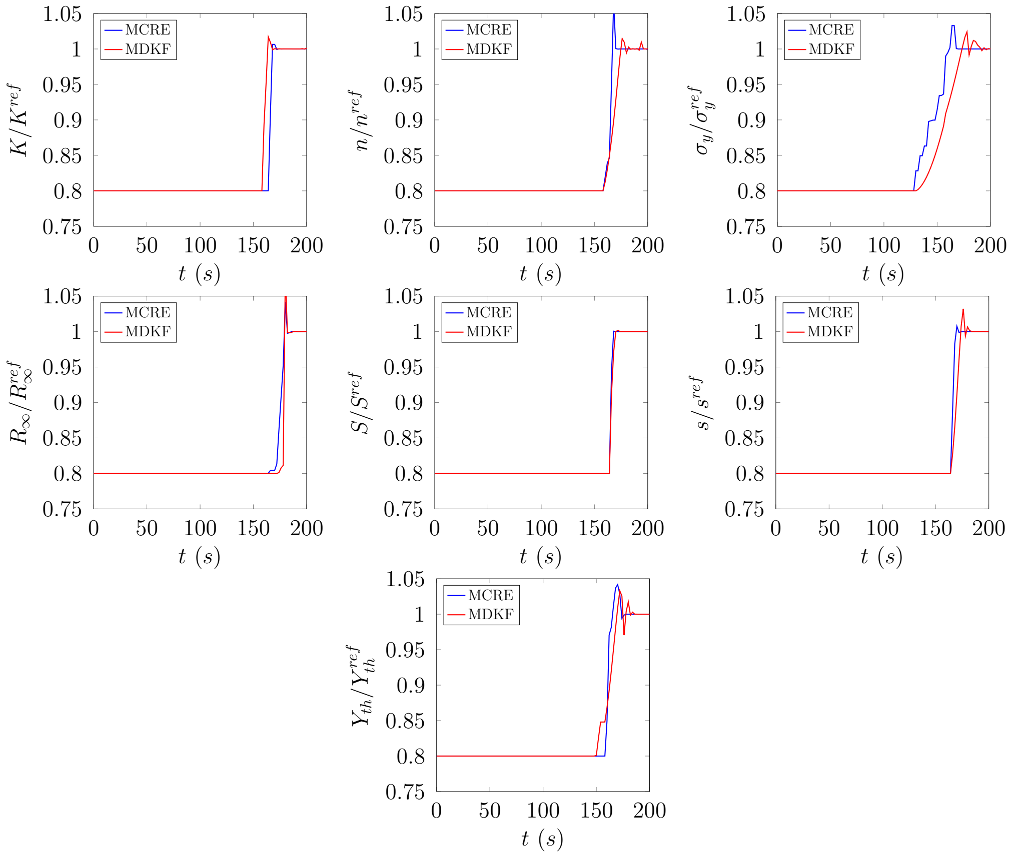

FIGURE 4 – Evolution of material parameters with respect to time

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Références

[1] L. Chamoin. *Merging advanced sensing techniques and simulation tools for future structural health monitoring technologies*, Project Repository Journal, 10, 124–127, 2021.

[2] S. Avril, M. Bonnet, A.S. Bretelle, M. Grédiac, F. Hild, P. Ienny, F. Latourte, D. Lemosse, S. Pagano, E. Pagnacco, F. Pierron. *Overview of Identification Methods of Mechanical Parameters Based on Full-field Measurements*, Experimental Mechanics, 48(4), 381-402, 2008.

[3] B. Marchand, L. Chamoin, C. Rey. *Parameter identification and model updating in the context of nonlinear mechanical behaviors using a unified formulation of the modified Constitutive Relation Error concept*, Computer Methods in Applied Mechanics and Engineering, 345, 1094-1113, 2019.

[4] H. N. Nguyen, L. Chamoin, C. Ha Minh. *mCRE-based parameter identification from full-field measurements : Consistent framework, integrated version, and extension to nonlinear material behaviors*, Computer Methods in Applied Mechanics and Engineering, 400, 115461, 2022.

[5] F. Chinesta, A. Ammar, A. Leygue, R. Keunings. *An overview of the proper generalized decomposition with applications in computational rheology*, Journal of Non-Newtonian Fluid Mechanics, 166(11), 578-592, 2011.

[6] M. Diaz and P.-É. Charbonnel, L. Chamoin. *A new Kalman filter approach for structural parameter tracking : Application to the monitoring of damaging structures tested on shaking-tables*, Mechanical Systems and Signal Processing, 182, 109529, 2023.

[7] M. Diaz and P.-É. Charbonnel, L. Chamoin. *Robust energy-based model updating framework for random processes in dynamics : Application to shaking-table experiments*, Computers & Structures, 264, 106746, 2022.

[8] S. Huang and P. Feissel, P. Villon. *Modified constitutive relation error : An identification framework dealing with the reliability of information*, Computer Methods in Applied Mechanics and Engineering, 311, 1-17, 2016.