

# Surface growth viewed as a phase transformation in coupled systems

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## Résumé —

We propose a 3D continuum framework for modeling surface growth in coupled systems, motivated by solid-state batteries. By distinguishing between the growing and conducting phases, we capture their distinct kinematics and derive jump conditions at the interface, which is material for the conducting phase but non-material for the accreting phase. We derive a thermodynamically consistent driving force conjugate to the growth velocity from the dissipation inequality. Analysis of a 1D model problem highlights the intrinsic coupling between accretion and the chemo-mechanical state of the system.

**Mots clés** — Surface growth, Phase transformation, Continuum thermodynamics.

## 1 Introduction

The development of Solid-State Batteries (SSBs) is hindered by the mechanical failure of the solid electrolyte (SE), often driven by the growth of lithium-metal dendrites [1]. Understanding the stress fields generated during lithium plating and stripping is critical for predicting inherently linked processes of the dendritic growth, the fracture of the SE, the stability of the anode/SE interface, and the void evolution within the anode [2]. Current modeling of stresses in metal anodes is largely restricted to bulk growth approaches [3], which fail to capture the fundamental surface-growth nature of lithium plating and stripping, or modeling of stresses in dendrites and SE without full consideration of stress state in the lithium-metal anode [4, 5].

We address the fundamental problem of a body undergoing surface growth coupled to a medium that conducts mass to its interface, deriving a thermodynamically consistent driving force conjugate to the growth velocity. Our work is inspired by the mechanics of solid-to-solid phase transformations and presents a general approach to model systems where surface growth is coupled to diffusion and mechanics. We consider a system consisting of two phases : a *growing* phase and a *conducting* phase (representing the lithium anode and the SE, respectively). The growing phase is a solid composed of bonded species "A"; these species are bonded within the bulk but can attach and detach only at the phase boundary. The conducting phase is a solid composed of species "B" which serves as a host allowing the diffusion of free species "A". Consequently, the growing phase undergoes accretion and ablation, while the conducting phase acts as a conduit for the transport of mass to the interface. To our knowledge, state-of-the-art surface growth theories have not addressed this specific coupled problem [6, 7]. The notable exception is the theory proposed in [8]; however, that framework treats a problem where diffusion occurs *inside* the accreting body and is limited to incompressible materials and rigid growth surface, a setting which is not applicable to SSBs.

## 2 Theory

### 2.1 Three configurations

Consider a body consisting of conducting and growing phases as outlined in Section 1. We assume the body is initially stress-free. Let  $\mathcal{B}_0$  denote this *initial configuration*, partitioned into the growing phase  $\mathcal{B}_0^g$  and the conducting phase  $\mathcal{B}_0^c$ . At any subsequent time  $t$ , we define a *reference configuration*  $\mathcal{B}$  (distinct from  $\mathcal{B}_0$ ) similarly divided into a growing phase  $\mathcal{B}^g$  and a conducting phase  $\mathcal{B}^c$ . The evolution

of the growing phase is characterized by the continuous attachment and detachment of material points at its boundary, altering the set of material points constituting  $\mathcal{B}^g$  (which is typical of surface growth problems [6]). Conversely, the conducting phase  $\mathcal{B}^c$  comprises a time-independent set of material points. To accommodate the geometric evolution of  $\mathcal{B}^g$  within the fixed domain of the reference configuration, the conducting phase undergoes a *fictitious displacement*. Thus, while the phase  $\mathcal{B}^g$  expands spatially, the material of the conducting phase is preserved rather than converted. The boundary motion is handled via a remapping of the reference configuration  $\mathcal{B}^c$ . Thus, we define three distinct configurations to describe

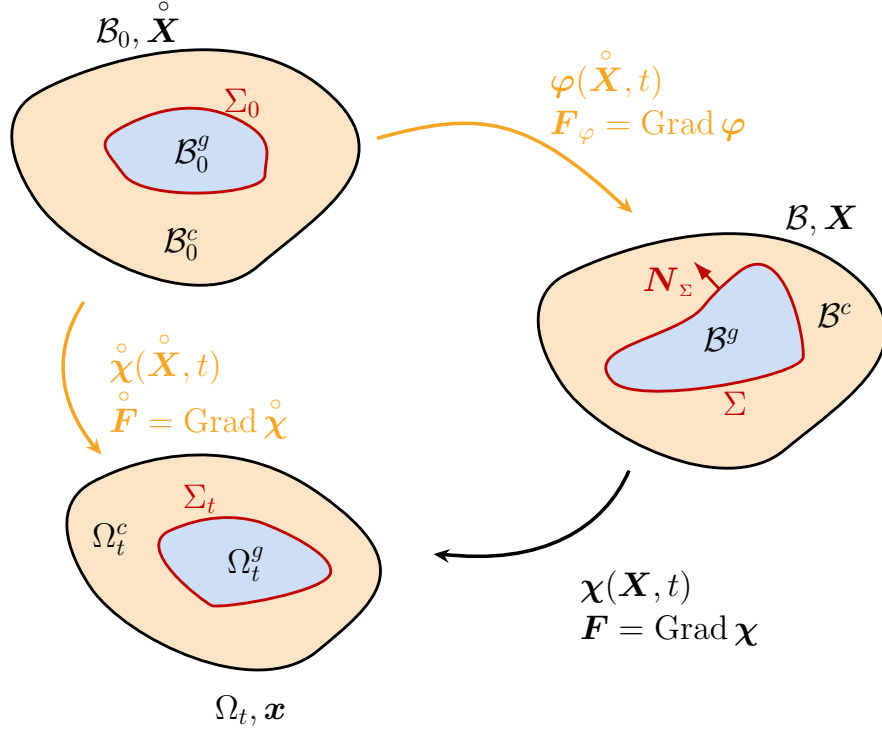


FIGURE 1 – The initial configuration  $\mathcal{B}_0$ , the reference configuration  $\mathcal{B}$  and the deformed configuration  $\Omega_t$ , with their associated phases and displacement fields (real and fictitious).

the system's evolution (as shown in Figure 1) :

1.  $\mathcal{B}_0$  : The *initial configuration* at  $t = 0$ , which is subdivided into  $\mathcal{B}_0^g$  and  $\mathcal{B}_0^c$ .
2.  $\mathcal{B}$  : The *reference configuration* at time  $t$ , which is subdivided into  $\mathcal{B}^g$  and  $\mathcal{B}^c$ . This configuration evolves to accommodate the surface accretion and ablation of  $\mathcal{B}^g$ , while the material for those processes is conducted through  $\mathcal{B}^c$ .
3.  $\Omega_t$  : The *actual deformed configuration* in ambient space subdivided into  $\Omega_t^c$  and  $\Omega_t^g$ .

Let  $\Sigma$  denote the phase boundary in  $\mathcal{B}$ , parametrized by  $\xi(\alpha_1, \alpha_2, t)$ . The phase boundary is a material boundary for the conducting phase, while it is a non-material boundary for the growing phase, which has profound consequences on the jump condition at the interface.

The conducting phase is deformed by a fictitious displacement  $\varphi$  mapping from  $\mathcal{B}_0^c$  to  $\mathcal{B}^c$ . While  $\varphi$  has to satisfy  $\varphi(\Sigma_0) = \Sigma$ , it is arbitrary in the bulk. Note that  $\varphi$  is not defined for the growing phase, as the set of material points at initial time and time  $t$  might not be the same in  $\mathcal{B}_0^g$  and  $\mathcal{B}^g$ . The deformation from the reference  $\mathcal{B}$  to the current configuration  $\Omega_t$  is described by the mapping  $\chi(\mathbf{X}, t)$  with the associated deformation gradient defined as  $\mathbf{F} = \text{Grad } \chi$ . Analogously, we denote by  $\overset{\circ}{\chi}(\overset{\circ}{\mathbf{X}}, t)$  the mapping of the conducting phase from the initial  $\mathcal{B}_0^c$  to the current  $\Omega_t^c$  configurations and by  $\overset{\circ}{\mathbf{F}}$  its deformation gradient.

Our goal is to formulate a set of governing equations on the reference configuration  $\mathcal{B}$ . The operators Div and Grad utilized in this paper are with respect to  $\mathbf{X}$ .

## 2.2 Kinematics

We present results under the assumption of normal growth and no-slip condition at the phase boundary, which mathematically is given by  $\partial_t \xi = \partial_t \varphi = v_\Sigma N_\Sigma$  on  $\Sigma$ , where  $N_\Sigma$  is the normal to  $\Sigma$  pointing into the conducting phase.

Inside the conducting phase  $\mathcal{B}^c$ , the velocity of a material point relative to the reference configuration is non-trivial due to the evolution of the fictitious displacement  $\varphi$ . This velocity  $v(\mathbf{X}, t)$  is given by :

$$v(\mathbf{X}, t) = \begin{cases} \partial_t \chi & \text{for } \mathbf{X} \in \mathcal{B}^g, \\ \partial_t \chi + \mathbf{F} \partial_t \varphi & \text{for } \mathbf{X} \in \mathcal{B}^c. \end{cases} \quad (1)$$

The standard Hadamard compatibility conditions hold, notably  $[[\partial_t \chi]] + [[\mathbf{F}]] \partial_t \xi = \mathbf{0}$ , however due to the fictitious displacement  $\varphi$  we have

$$[[v]] = \mathbf{F}^- v_\Sigma N_\Sigma; \quad (2)$$

where the jump is evaluated at the interface  $\Sigma$  between the two phases and  $\mathbf{F}^-$  is the deformation gradient evaluated on the side of the growing phase  $\mathcal{B}^g$ .

## 2.3 Mass balance

We assume that there is no diffusion in the growing phase, while the conducting phase allows for diffusion.

- **In the bulk :** The local mass balance for growing material in the conducting phase  $\mathcal{B}^c$  is governed by the diffusion equation :

$$\frac{d(J_\varphi \rho)}{dt} + J_\varphi \text{Div } \mathbf{j} = 0, \quad (3)$$

where  $\rho$  is the density of the diffusing free "A" species in the reference configuration,  $\mathbf{j}$  is their mass flux and  $J_\varphi = \det \mathbf{F}_\varphi$ .

- **At the interface :** The localization of the mass balance equation on the moving interface  $\Sigma$  yields the jump condition :

$$\rho_0^g v_\Sigma + j_N = 0. \quad (4)$$

Here,  $\rho_0^g$  is the natural density of the growing phase  $\mathcal{B}^g$ , and  $j_N = \mathbf{j} \cdot \mathbf{N}_\Sigma$  is the normal flux. This equation directly couples the growth velocity  $v_\Sigma$  to the mass supply.

## 2.4 Mechanical equilibrium

Neglecting inertial effects and body forces (quasi-static assumption), the mechanical equilibrium in the reference configuration  $\mathcal{B}$  requires :

$$\text{Div } \mathbf{P} = \mathbf{0} \quad \text{in } \mathcal{B}, \quad (5)$$

where  $\mathbf{P}$  is the First Piola-Kirchhoff stress tensor. Across the interface  $\Sigma$ , mechanical equilibrium requires the continuity of traction :

$$[[\mathbf{P}]] \mathbf{N}_\Sigma = \mathbf{0}. \quad (6)$$

## 2.5 Free-energy imbalance

Assuming isothermal conditions and denoting by  $\psi$  the Helmholtz free-energy per unit volume of the reference configuration, the free-energy imbalance for a material volume  $\mathcal{P} \subset \mathcal{B}$  in the reference configuration reads as :

$$\frac{d}{dt} \int_{\mathcal{P}} \psi dV \leq \int_{\partial \mathcal{P}} \mathbf{v} \cdot (\mathbf{P} \mathbf{N}) dS - \int_{\partial \mathcal{P}^c} \mu \mathbf{j} \cdot \mathbf{N} dS, \quad (7)$$

where  $\mu$  is the chemical potential of the diffusing species and  $\partial \mathcal{P}^c = \partial \mathcal{P} \cap \mathcal{B}^c$ . Note that the material volume may be time-dependent if it contains material points of the conducting phase  $\mathcal{B}^c$ , since the conducting phase is undergoing fictitious displacements  $\varphi$ . Localizing (7) :

— **In the growing phase  $\mathcal{B}^g$**  : Assuming elastic constitutive behavior we obtain

$$\frac{\partial \psi}{\partial \mathbf{F}} = \mathbf{P}, \quad (8)$$

The time derivative in (7) can be brought directly inside the integral, since  $\mathcal{P}$  is invariant in time.

— **In the conducting phase  $\mathcal{B}^c$**  : Assuming elastic constitutive behavior and Fick's law of diffusion, we obtain

$$\frac{\partial \psi}{\partial \mathbf{F}} = \mathbf{P}, \quad \frac{\partial \psi}{\partial \rho} = \mu, \quad \mathbf{j} = -J_{\varphi}^{-1} \mathbf{F}_{\varphi} \overset{\circ}{\mathbf{M}} \mathbf{F}_{\varphi}^T \text{Grad} \mu, \quad (9)$$

where  $\overset{\circ}{\mathbf{M}}$  is the mobility of species "A" in the initial configuration of the conducting phase. These constitutive relations are obtained by first pulling back  $\mathcal{P}$  to the initial configuration  $\mathcal{B}_0^c$ , where it is invariant in time and then by pushing all the quantities forward by  $\varphi$  to the reference configuration.

— **At the boundary  $\Sigma$**  :

$$v_{\Sigma}(\psi^{-} - \mathbf{F}^{-} \mathbf{N}_{\Sigma} \cdot \langle \mathbf{P} \rangle \mathbf{N}_{\Sigma} - \mu^{+} \rho_0^g) \leq 0 \quad (10)$$

where + and – refer to quantities evaluated on the sides of the conducting and growing phases, respectively. To obtain this result, the integrals in (7) are split into the part in  $\mathcal{B}^g$  and  $\mathcal{B}^c$  and the appropriate Reynolds transport theorem and divergence theorem is used. From which we obtain the driving force of surface accretion conjugate to the growth velocity  $v_{\Sigma}$  :

$$f = -\psi^{-} + \mathbf{F}^{-} \mathbf{N}_{\Sigma} \cdot \langle \mathbf{P} \rangle \mathbf{N}_{\Sigma} + \mu^{+} \rho_0^g \quad (11)$$

For comparison, the classical driving force in a diffusion-driven solid-to-solid phase transformation is  $[\psi - \mu \rho] - \langle \mathbf{P} \rangle \mathbf{N}_{\Sigma} \cdot [\mathbf{F}] \mathbf{N}_{\Sigma}$  [9]. Note that our driving force in (11) does not involve jumps over the interface (a somewhat similar expression is obtained in [8], but in a distinctly different and more restrictive setting). This is due to the nature of the surface  $\Sigma$ , which represents a material boundary for the conducting phase and a non-material boundary for the growing phase.

### 3 1D Example

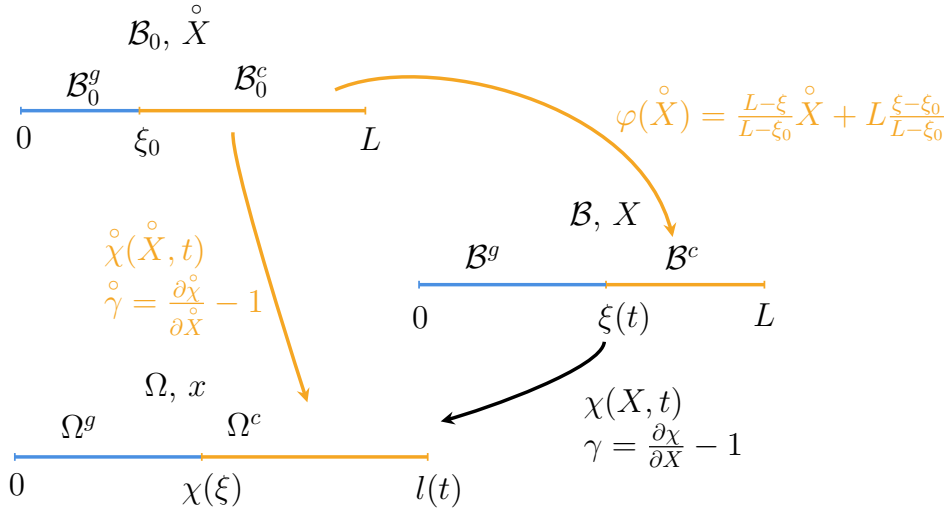


FIGURE 2 – Schematic of the 1D problem showing the initial reference configuration  $\mathcal{B}_0$ , the current reference configuration  $\mathcal{B}$ , and the deformed configuration  $\Omega_t$ , along with the fictitious and real mappings.

To illustrate the framework, we examine a one-dimensional problem. The initial configuration is defined as  $\mathcal{B}_0 = [0, L]$ , partitioned into a growing phase  $\mathcal{B}_0^g = [0, \xi_0]$  and a conducting phase  $\mathcal{B}_0^c = (\xi_0, L]$  for some initial interface position  $\xi_0 \in (0, L)$ . At time  $t$ , the reference configuration remains  $\mathcal{B} = [0, L]$ ,

but the phase boundary evolves to a new position  $\xi(t)$ , yielding  $\mathcal{B}^s = [0, \xi]$  and  $\mathcal{B}^c = (\xi, L]$  (see Figure 2). We prescribe the fictitious displacement of the conducting phase,  $\varphi : \mathcal{B}_0^c \rightarrow \mathcal{B}^c$ , to be a linear mapping that accommodates the interface motion :

$$\varphi(\overset{\circ}{X}) = \frac{L - \xi}{L - \xi_0} \overset{\circ}{X} + L \frac{\xi - \xi_0}{L - \xi_0}. \quad (12)$$

This ensures  $\varphi(\xi_0) = \xi$  and  $\varphi(L) = L$ . Mechanically, the body is clamped at  $X = 0$  and stress at  $X = L$  is specified :  $\sigma(L) = \sigma_0$ . Chemically, the density of the diffusing lithium is fixed at the boundary  $L$ , such that  $\overset{\circ}{\rho}(L) = \rho_L$ , corresponding to a density in the initial configuration.

The growing phase (Lithium) is modeled as a linear elastic solid :

$$\Psi = \Psi_0^{Li} + \frac{1}{2} E_{Li} \gamma^2, \quad (13)$$

$$\sigma = E_{Li} \gamma, \quad (14)$$

where  $E_{Li}$  is the Young's modulus and  $\gamma = \partial_X \chi - 1$  is the strain. The conducting phase (SE) is modeled as a linear chemo-elastic body. Following [10], the constitutive relations in the initial configuration  $\mathcal{B}_0^c$  are :

$$\overset{\circ}{\Psi}(\overset{\circ}{\gamma}, \overset{\circ}{\rho}) = \overset{\circ}{\mu}_0 \overset{\circ}{\rho} + \frac{1}{2} \overset{\circ}{E}_{SE} \overset{\circ}{\gamma}^2 + \frac{1}{2} \overset{\circ}{\Lambda} \overset{\circ}{\rho}^2 + \overset{\circ}{\Psi}_0, \quad (15)$$

$$\sigma = \overset{\circ}{E}_{SE} \overset{\circ}{\gamma}, \quad \overset{\circ}{\mu} = \overset{\circ}{\mu}_0 + \overset{\circ}{\Lambda} \overset{\circ}{\rho}, \quad j = -\overset{\circ}{M}_{Li} \frac{\partial \overset{\circ}{\mu}}{\partial \overset{\circ}{X}}, \quad (16)$$

where  $\overset{\circ}{\gamma} = \partial_{\overset{\circ}{X}} \overset{\circ}{\chi} - 1$ ,  $\overset{\circ}{\rho}$  is the density per unit volume of the initial configuration of the free lithium diffusing in the SE;  $\overset{\circ}{E}_{SE}$  and  $\overset{\circ}{\Lambda}$  are the elastic and chemical moduli, and  $\overset{\circ}{M}_{Li}$  is the mobility . We do not explicitly introduce chemo-mechanical coupling into the constitutive equations of the SE, yet in the resultant driving force (26) both the stress-state and chemical parameters are crucial. One can also formulate these equation in the reference configuration as in (9) :

$$\Psi(\gamma, \rho) = \mu_0 \rho + \frac{1}{2} E_{SE} \gamma^2 + (E_{SE} - \overset{\circ}{E}_{SE}) \gamma + \frac{1}{2} \Lambda \rho^2 + \Psi_0, \quad (17)$$

$$\sigma = E_{SE} (1 + \gamma) - \overset{\circ}{E}_{SE}, \quad \mu = \mu_0 + \Lambda \rho, \quad j = -M_{Li} \frac{\partial \mu}{\partial X}, \quad (18)$$

where the effective moduli scale as  $E_{SE} = \overset{\circ}{E}_{SE} \varphi'$ ,  $\Lambda = \overset{\circ}{\Lambda} \varphi'$ , and  $M_{Li} = \overset{\circ}{M}_{Li} \varphi'$  with  $\varphi' = \partial_{\overset{\circ}{X}} \varphi$ .

Given the traction boundary condition  $\sigma(L) = \sigma_0$  and the equilibrium condition  $\partial_X \sigma = 0$ , the strain in the growing phase is  $\gamma = \sigma_0 / E_{Li}$  and in the conducting phase is  $\overset{\circ}{\gamma} = \sigma_0 / \overset{\circ}{E}_{SE}$ . Consequently, the deformation is :

$$\chi(X, t) = \left(1 + \frac{\sigma_0}{E_{Li}}\right) X \quad \text{in } \mathcal{B}^s, \quad \text{and} \quad (19)$$

$$\overset{\circ}{\chi}(\overset{\circ}{X}, t) = \left(1 + \frac{\sigma_0}{\overset{\circ}{E}_{SE}}\right) \left(\overset{\circ}{X} - \xi_0\right) + \left(1 + \frac{\sigma_0}{E_{Li}}\right) \xi(t) \quad \text{in } \mathcal{B}_0^c. \quad (20)$$

This yields the velocity field :

$$v = \begin{cases} 0 & \text{in } \mathcal{B}^s, \\ \left(1 + \frac{\sigma_0}{E_{Li}}\right) \partial_t \xi & \text{in } \mathcal{B}^c. \end{cases} \quad (21)$$

The physical velocity jump is given by (2) :  $\llbracket v \rrbracket = \left(1 + \frac{\sigma_0}{E_{Li}}\right) \partial_t \xi$  (which is directly visible from (21)). For  $\sigma_0 = 0$  we obtain rigid body motion : the lithium bar is not moving and the SE is translating with the growth velocity  $\partial_t \xi$ . The thermodynamic driving force  $f$  simplifies to :

$$f = \left(-\Psi_0^{Li} + \overset{\circ}{\mu}_0 \rho_0\right) + \left(1 + \frac{\sigma_0}{2E_{Li}}\right) \sigma_0 + \overset{\circ}{\Lambda} \overset{\circ}{\rho}^+ \rho_0, \quad (22)$$

where  $\rho_0$  is the mass density of stress-free lithium. The first term is a constant linking intrinsic properties of the two phases, the second and third terms represent the mechanical and chemical contribution to the driving force, respectively. The system is in thermodynamic equilibrium ( $f = 0$ ) when the density of free lithium in the SE at the interface between phases reaches the equilibrium value :

$$\overset{\circ}{\rho}^{eq} = \frac{\Psi_0^{Li} - \overset{\circ}{\mu}_0 \rho_0 - \left(1 + \frac{\sigma_0}{2E_{Li}}\right) \sigma_0}{\rho_0 \overset{\circ}{\Lambda}}. \quad (23)$$

We seek steady-state solutions ( $\partial_t \overset{\circ}{\rho} = 0$ ) assuming a linear kinetic relation  $\partial_t \xi = \beta f$ . This implies a linear profile of the density of free lithium (per unit volume of the initial configuration of the SE)  $\overset{\circ}{\rho}(\overset{\circ}{X}) = c\overset{\circ}{X} + d$ , where . Using the boundary conditions  $\overset{\circ}{\rho}(L) = \rho_L$  and the mass balance at the interface  $j(\xi) = -\partial_t \xi \rho_0$  (from (4)), we solve for the coefficients  $c$  and  $d$  :

$$c = \frac{\beta \rho_0 f}{\overset{\circ}{M}_{Li} \overset{\circ}{\Lambda}}, \text{ and} \quad (24)$$

$$d = \rho_L - \frac{\beta \rho_0 f}{\overset{\circ}{M}_{Li} \overset{\circ}{\Lambda}} L. \quad (25)$$

with the driving force given by

$$f = \frac{\overset{\circ}{M}_{Li} \left( \sigma_0 + \frac{\sigma_0^2}{2E_{Li}} + \overset{\circ}{\mu}_0 \rho_0 + \overset{\circ}{\Lambda} \rho_0 \rho_L - \Psi_0^{Li} \right)}{\overset{\circ}{M}_{Li} + \beta(L - \xi_0) \rho_0^2} = \frac{\overset{\circ}{M}_{Li} \overset{\circ}{\Lambda} \rho_0}{\overset{\circ}{M}_{Li} + \beta(L - \xi_0) \rho_0^2} (\rho_L - \overset{\circ}{\rho}^{eq}). \quad (26)$$

Therefore, the driving force depends on both the mechanical stress and the chemical parameters. This

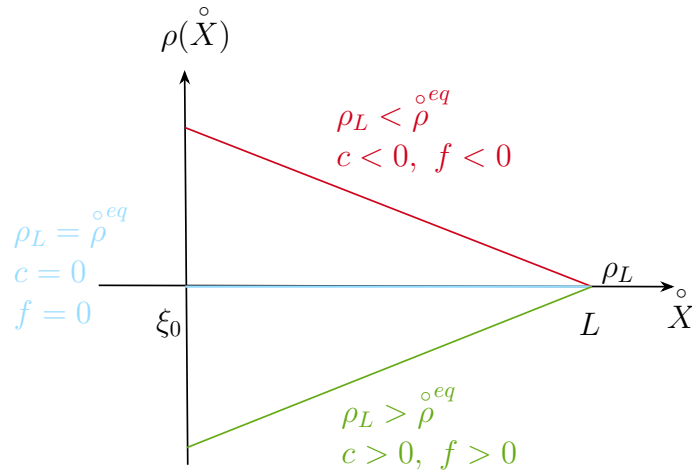


FIGURE 3 – Schematic of the three possible solutions of the density of free lithium diffusing in the initial configuration of the SE,  $\mathcal{B}_0^c = [\xi_0, L]$ .

highlights the intrinsic coupling between surface growth and the stress state of the system, even in the absence of direct chemo-mechanical coupling in the bulk constitutive laws. We observe that the density gradient of free lithium diffusing in the SE is directly proportional to the driving force and the difference between  $\rho_L$  and  $\overset{\circ}{\rho}^{eq}$  ( $c \propto f \propto (\rho_L - \overset{\circ}{\rho}^{eq})$ ). Thus, three distinct solutions are found, as illustrated in Figure 3 : (i) ablation ( $f < 0$ ) when  $\rho_L < \overset{\circ}{\rho}^{eq}$  ; (ii) equilibrium ( $f = 0$ ) when  $\rho_L = \overset{\circ}{\rho}^{eq}$  , where the density is uniform and the interface is stationary ; and (iii) accretion ( $f > 0$ ) when  $\rho_L > \overset{\circ}{\rho}^{eq}$  .

Physically, the density is bounded by  $0 \leq \overset{\circ}{\rho} \leq \rho_{max}$ . Parameter sets yielding steady-state solutions outside these bounds (e.g.,  $\overset{\circ}{\rho}(\xi_0) < 0$  or  $\overset{\circ}{\rho}(\xi_0) > \rho_{max}$ ) correspond to regimes where a physical steady

state is unattainable. Finally, we examine the regulatory influence of the imposed stress. Sufficient compressive stress ( $\sigma_0 < 0$ ) decreases  $f$  and may suppress the reaction entirely. However, it can also act as a stabilizer : consider ablation, where a high value of  $\rho_L$  (for  $\sigma_0 = 0$ ) causes the density of the diffusing lithium at the phase boundary  $\dot{\rho}(\xi_0)$  to be larger than  $\rho_{max}$ , which would correspond to an unstable regime. Imposing a compressive stress  $\sigma_0 < 0$  reduces the density at the phase interface  $\dot{\rho}(\xi_0)$ , potentially restoring a steady-state solution to the system.

## 4 Conclusions

We have presented a continuum framework for modeling surface growth in coupled systems, motivated by the phenomenology of solid-state batteries. A central feature of this theory is the kinematic decomposition of the system into conducting and growing phases. While the set of material points in the conducting phase is preserved, the growing phase undergoes surface accretion and ablation. Crucially, the phase boundary  $\Sigma$  acts as a material surface for the conducting phase but as a non-material surface for the growing phase. This dual nature distinguishes our approach from standard solid-to-solid phase transformation models, where the interface is typically non-material for both phases (e.g., coherent phase boundaries), as well as from classical surface accretion theories, where the growing body is not coupled to a deformable environment that allows diffusion.

We derived jump conditions based on kinematic compatibility and the balance of linear momentum. By rigorously exploiting the dissipation inequality, we obtained a thermodynamic driving force  $f$  conjugate to the growth velocity. The expression for  $f$  explicitly couples the mechanical work performed by the conducting host, the elastic energy of the accreted material, and the chemical potential of the diffusing species. This driving force differs distinctively from the classical force found in diffusion-driven phase transformations, reflecting the specificity of the accretion/ablation processes.

Finally, we analyzed a one-dimensional prototype problem utilizing linear constitutive laws. We derived the closed-form steady-state solution and the associated driving force. The analysis demonstrates that, even in the absence of a direct chemo-mechanical coupling in the constitutive laws, the driving force as well as the density of diffusing lithium in the SE depend on both the mechanical stress of the system as well as on the chemical parameters.

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